

Limits and Continuity

Grade: «grade»

Subject: «subject»

Date: «date»

1 Answer?

A calculator may not be used on the following questions.

1. Evaluate the limit, if it exists: $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2 - x}$.
- (A) 5
 - (B) 3
 - (C) -3
 - (D) -5
 - (E) The limit does not exist.

2 Answer?

2. Evaluate the limit, if it exists: $\lim_{x \rightarrow 9} \frac{\sqrt{x-5}-2}{x-9}$.

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) 1

(D) 0

(E) The limit does not exist.

3 Answer?

3. Evaluate the limit, if it exists: $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$.

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) 1

(D) -1

(E) The limit does not exist.

4 Answer?

4. Evaluate the limit, if it exists: $\lim_{x \rightarrow 1} \frac{\tan^{-1} x}{\sin^{-1} x + 1}$.

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{\pi}{2}$

(E) $\frac{\pi}{2\pi + 4}$

5 Answer?

Estimate the limit, if it exists: $\lim_{x \rightarrow 3} f(x)$ where $f(x)$ represented by the given graph:

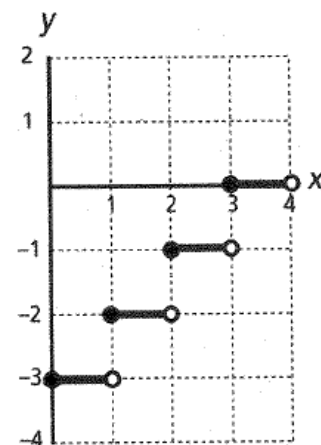
(A) 0

(B) -1

(C) 3

(D) 1

(E) The limit does not exist.



6 Answer?

6. Given the function

$$f(x) = \begin{cases} \sin 2x, & x \leq \pi \\ 2x + k, & x > \pi \end{cases}$$

what value of k will make this piecewise function continuous?

- (A) -2π
- (B) $-\pi$
- (C) 0
- (D) π
- (E) 2π

7 Answer?

7. Find the limit, if it exists: $\lim_{x \rightarrow 0} x \left(e^x + \frac{1}{x} \right)$.

- (A) 0
- (B) 1
- (C) 2
- (D) The limit does not exist.
- (E) None of these

8 Answer?

8. Identify the vertical asymptotes for $f(x) = \frac{x^2 + 3x - 4}{x^2 + x - 2}$.

- (A) $x = -2, x = 1$
- (B) $x = -2$
- (C) $x = 1$
- (D) $y = -2, y = 1$
- (E) $y = -2$

9 Answer?

9. If $p(x)$ is a continuous function on the closed interval $[1, 3]$, with $p(1) \leq K \leq p(3)$ and c is in the closed interval $[1, 3]$, then which of the following statements must be true?

- (A) $p(c) = \frac{p(3) + p(1)}{2}$
- (B) $p(c) = \frac{p(3) - p(1)}{2}$
- (C) There is at least one value c such that $p(c) = K$.
- (D) There is only one value c such that $p(c) = K$.
- (E) $c = 2$

10 Answer?

10. How many vertical asymptotes exist for the function

$$f(x) = \frac{1}{2 \sin^2 x - \sin x - 1} \text{ in the open interval } 0 < x < 2\pi?$$

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

11 Answer?

| | | | | |
|--------|---|---|---|---|
| x | 1 | 2 | 3 | 4 |
| $f(x)$ | 4 | 2 | 3 | 1 |
| $g(x)$ | 2 | 3 | 1 | 4 |

11. Selected values for continuous functions $f(x)$ and $g(x)$ are given in the table above. $\lim_{x \rightarrow 3} \frac{f(g(x))}{g(f(x))} =$

- (A) $\frac{1}{4}$
- (B) $\frac{1}{3}$
- (C) 1
- (D) 3
- (E) 4

12 Answer?

12. $\lim_{x \rightarrow \infty} \frac{\sin x}{e^x + \cos x} =$

(A) -1

(B) 0

$\frac{1}{e}$

(C) e

(D) 1

(E) The limit does not exist.

13 Answer?

13. For what value of k is the function $f(x) = \begin{cases} \frac{2x^2 + 5x - 3}{x^2 - 9}, & x \neq -3 \\ k, & x = -3 \end{cases}$

continuous at $x = -3$?

(A) $-\frac{7}{6}$

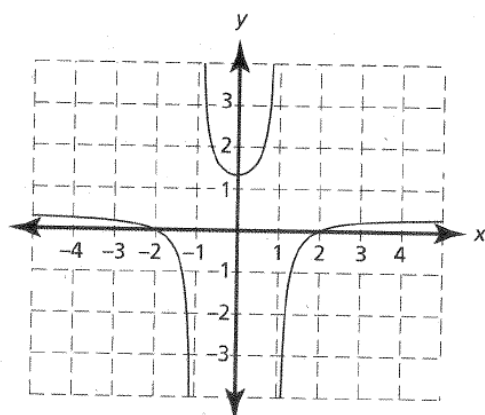
(B) $-\frac{5}{6}$

(C) 0

(D) $\frac{5}{6}$

(E) $\frac{7}{6}$

14 Answer?



14. The function $g(x)$ is shown in the graph above and is of the form

$$g(x) = \frac{x^2 + a}{bx^2 - 3}.$$

Which of the following could be the values of the

constants a and b ?

- (A) $a = -2$, $b = -1$
- (B) $a = -2$, $b = -3$
- (C) $a = -4$, $b = 3$
- (D) $a = -4$, $b = -3$
- (E) $a = 4$, $b = 3$

15 Answer?

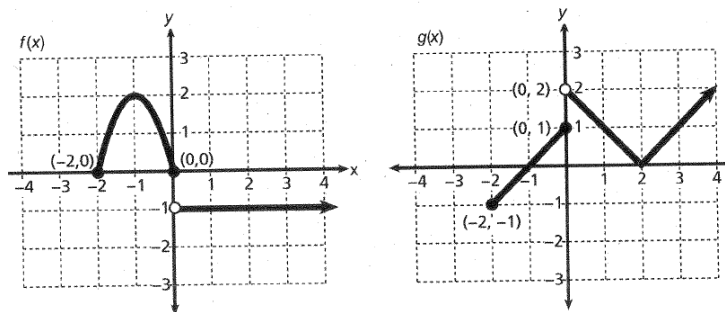
15. Which of the following statements is true?

- (A) $\lim_{x \rightarrow 3} \log_3 x = 2$
- (B) $\lim_{x \rightarrow 0^+} \log_3 x$ does not exist.
- (C) $\lim_{x \rightarrow -\infty} e^x$ does not exist.
- (D) $\lim_{x \rightarrow -\frac{\pi}{2}} \csc x = 1$
- (E) $\lim_{x \rightarrow 1} e^{x-1} = 0$

FREE-RESPONSE QUESTION

A calculator may not be used for this question.

1. Use the graphs of $f(x)$ and $g(x)$ given below to answer the following questions:



- Is $f[g(x)]$ continuous at $x = 0$? Explain why or why not.
- Is $g[f(x)]$ continuous at $x = 0$? Explain why or why not.
- What is $\lim_{x \rightarrow 0} f[g(x)]$? Explain your reasoning.
- If $h(x) = \begin{cases} f(x) + g(x), & -2 \leq x \leq 0 \\ k + g(x)f(x), & x > 0 \end{cases}$, what is k so that $h(x)$ is continuous at $x = 0$?

Derivative

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Calculators may not be used on this part of the examination.

1 Answer?

1. What does the limit statement $\lim_{x \rightarrow 1} \frac{\ln(x+1) - \ln 2}{x-1}$ represent?
- (A) 0
 - (B) $\frac{d}{dx}[\ln(x+1)]$
 - (C) $f'(1)$, if $f(x) = \ln(x+1)$
 - (D) 1
 - (E) The limit does not exist.

2 Answer?

2. Find the derivative of the function $y = \frac{4}{x^3}$.
- (A) $-4x^2$
 - (B) $-\frac{12}{x^2}$
 - (C) $\frac{12}{x^2}$
 - (D) $\frac{12}{x^4}$
 - (E) $-\frac{12}{x^4}$

3 Answer?

3. Find $\frac{dy}{dx}$ if $3xy = 4x + y^2$.

(A) $\frac{4-3y}{2y-3x}$

(B) $\frac{3x-4}{2x}$

(C) $\frac{3y-x}{2}$

(D) $\frac{3y-4}{2y-3x}$

(E) $\frac{4+3y}{2y+3x}$

4 Answer?

4. Find $\frac{dy}{dx}$ for $e^{x+y} = y$.

(A) $\frac{e^{x+y}}{(1-e^{x+y})}$

(B) $\frac{e^{x+y}}{(1+e^{x+y})}$

(C) $\frac{e^{x+y}}{(e^{x+y}-1)}$

(D) e^{x+y}

(E) $2e^{x+y}$

5 Answer?

5. If the n th derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(7)}$ is the same as
- (A) y
 - (B) $\frac{dy}{dx}$
 - (C) $\frac{d^2y}{dx^2}$
 - (D) $\frac{d^3y}{dx^3}$
 - (E) None of these

6 Answer?

6. Find the second derivative of $f(x)$ if $f(x) = (2x + 3)^4$.
- (A) $4(2x + 3)^3$
 - (B) $8(2x + 3)^3$
 - (C) $12(2x + 3)^2$
 - (D) $24(2x + 3)^2$
 - (E) $48(2x + 3)^2$

7 Answer?

A calculator may be used for any of the following multiple-choice questions.

7. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.

- (A) $8\sin(3x)$
- (B) $24\sin(3x)$
- (C) $8\sin(3x)\cos(3x)$
- (D) $12\sin(3x)\cos(3x)$
- (E) $24\sin(3x)\cos(3x)$

8 Answer?

If $\ln y = (\ln x)^2 + 2$, find $\frac{dy}{dx}$ in terms of x and y .

- (A) $y \left[2\ln(x) + \frac{1}{x} \right]$
- (B) $y \left[\left(\frac{2}{x} \right) \ln(x) \right]$
- (C) $\left(\frac{2}{x} \right) \ln(x)$
- (D) $\frac{2(\ln x)}{x} + 2$
- (E) $y \left[\frac{2(\ln x)}{x} + 2 \right]$

9 Answer?

If $f(2) = -3$, $f'(2) = \frac{3}{4}$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

(A) $y - 2 = \frac{-3}{4}(x + 3)$

(B) $y + 2 = \frac{-3}{4}(x - 3)$

(C) $y - 2 = \frac{-4}{3}(x + 3)$

(D) $y + 2 = \frac{4}{3}(x - 3)$

(E) $y - 2 = \frac{4}{3}(x + 3)$

10 Answer?

For what positive value of x does the tangent line to the curve $y = \ln(1 - x)$ intersect the y -axis at the point $(0, 2)$?

(A) 0.382

(B) 0.547

(C) 0.667

(D) 0.722

(E) 0.778

11 Answer?

Calculators may not be used for this part of the examination.

For what values of a and c is the piecewise function

$$f(x) = \begin{cases} ax^2 + \sin x, & x \leq \pi \\ 2x - c, & x > \pi \end{cases} \text{ differentiable?}$$

- (A) $a = \frac{3\pi}{2}$ and $c = \frac{\pi}{2}$
 (B) $a = \frac{3}{2\pi}$ and $c = \frac{7\pi}{2}$
 (C) $a = \frac{3}{2\pi}$ and $c = -\frac{\pi}{2}$
 (D) $a = \frac{3}{2\pi}$ and $c = \frac{\pi}{2}$
 (E) $a = \frac{3\pi}{2}$ and $c = \frac{2}{\pi}$

12 Answer?

If $y = \tan^{-1}(x^2 + 3x)$, then $\frac{dy}{dx} =$

- (A) $\frac{1}{1 + (x^2 + 3x)^2}$
 (B) $\frac{1}{x^2 + 3x + 1}$
 (C) $\frac{2x + 3}{1 + (x^2 + 3x)^2}$
 (D) $\frac{2x + 3}{(x^2 + 3x)^2}$
 (E) $\frac{x^2 + 3x}{1 + (x^2 + 3x)^2}$

13 Answer?

| x | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|-----|--------|--------|---------|---------|
| 1 | 3 | 1 | -2 | 4 |
| 2 | 5 | 3 | 1 | -4 |
| 3 | 2 | 1 | -2 | 1 |
| 4 | 4 | -3 | 2 | -1 |

Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are given in the table above. If $p(x) = x \cdot f(x) - g(3x - 2)$, then

$p'(2) =$

- (A) 11
- (B) 10
- (C) 8
- (D) 6
- (E) 4